PHYSICAL REVIEW E VOLUME 48, NUMBER 5 NOVEMBER 1993

Rotational invariance and the theory of directed nematic polymers

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(Received 9 June 1993)

The consequences of rotational invariance in a recent theory of fluctuations in dilute nematic polymers are explored. A correct rotationally invariant free energy ensures that anomalous couplings are not generated in a one-loop renormalization-group calculation.

PACS number(s): 36.20.Ey, 05.40.+j

I. INTRODUCTION AND SUMMARY

In this Brief Report we refine an earlier theory of nematic polymers [1]. In that paper a renormalization-group calculation was performed in the dilute-polymer limit which reproduced the anomalous, logarithmic wandering predicted by de Gennes [2].

We analyzed the model defined by the grand-canonical partition function:

$$\mathcal{Z}_{\rm gr} = \int \mathcal{D}\psi\, \mathcal{D}\psi^*\, \mathcal{D}\delta\vec{n}\, \exp\{-S[\psi,\psi^*,\delta\vec{n}]\}\,, \qquad (1)$$

where the action was broken into three parts,

$$\begin{aligned} S[\psi,\psi^*,\delta\vec{n}] &= \int d^d r \int dz\, \left[\psi^*\left(\partial_z - D\nabla_\perp^2 - \bar{\mu}\right)\psi + v|\psi|^4\right] \\ &\quad + \frac{\lambda}{2}\int d^d r \int dz\, \delta\vec{n}\cdot(\psi^*\nabla_\perp\psi - \psi\nabla_\perp\psi^*) \\ &\quad + F_n[\delta\vec{n}]/k_{\rm B}T \qquad (2) \end{aligned}$$

and

$$F_n = \frac{1}{2}\int d^2 r \int dz \left[K_1(\nabla_\perp\cdot\delta\vec{n})^2 + K_2(\nabla_\perp\times\delta\vec{n})^2 + K_3(\partial_z\delta\vec{n})^2\right], \qquad (3)$$

where ψ is the "boson" order parameter describing the directed polymers and $\delta\vec{n}$ is a d-dimensional vector which describes director fluctuations in the nematic matrix.

We performed a renormalization-group calculation near $\bar{\mu} = 0$ and found logarithmic corrections to mean-field theory in the critical dimension $d_c = d + 1 \equiv 2 + 1$. We show here that an additional $|\psi|^2\delta\vec{n}^2$ term, which appears to be generated at one-loop order, is canceled by additional coupling required by rotational invariance. We derived Eq. (1) starting with a rotationally invariant theory by choosing a broken-symmetry direction for the nematic (which we take to be the z axis), and expanding to quadratic order in fields. If we consider the nematic as an external, nonfluctuating field, then Eq. (1) should be invariant under the following transformation:

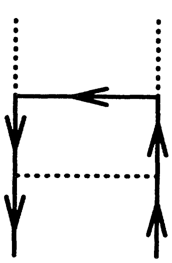

FIG. 1. One-loop graph in original model which generates a term $|\psi|^2\delta\vec{n}^2$. It is finite in $2+1$ dimensions. The dotted lines represent nematic field propagators. The solid lines represent boson field propagators.

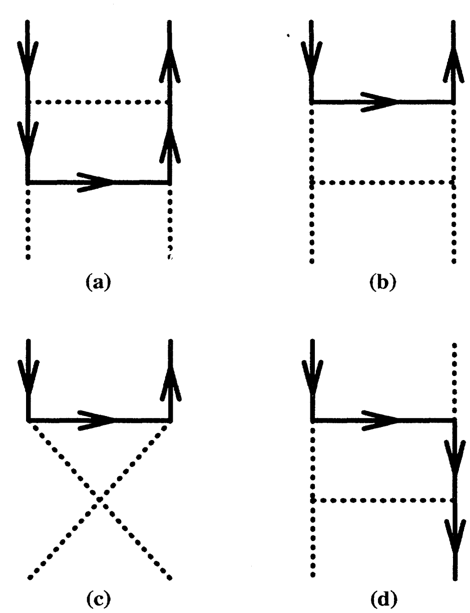

FIG. 2. Graphs which all contribute to $|\psi|^2\delta\vec{n}^2$ in a rotationally invariant theory. Note that they are all $O(\lambda^4)$. These graphs cancel among each other to prevent such a term.

$$\begin{aligned} \delta\vec{n}'(\vec{r}\,',z') &= \delta\vec{n}(\vec{r},z)+\vec{h}, \\ \psi'(\vec{r}\,',z') &= \psi(\vec{r},z), \\ \vec{r}\,' &= \vec{r}+\lambda\vec{h}z, \\ z' &= z. \end{aligned} \tag{4}$$

In the limit of small $\vec{h}$, this affine change of variables is equivalent to rotating the system by an amount $\vec{h}$. With this change of coordinates, $\partial_{z'}=\partial_z-\lambda\vec{h}\cdot\nabla_\perp$, $\nabla'_\perp=\nabla_\perp$ and $d^dr'dz'=d^dr\,dz$, and the part of the action S_1 which depends on ψ becomes S_1',

$$\begin{aligned} S_1'[\psi',\delta\vec{n}'] &= \int d^dr'dz'\left[\psi^{*\prime}\left[\partial_z'-D(\nabla'_\perp)^2-\bar{\mu}\right]\psi'+v|\psi'|^4+\frac{\lambda}{2}\delta\vec{n}'\cdot(\psi^{*\prime}\nabla'_\perp\psi'-\psi'\nabla'_\perp\psi^{*\prime})\right] \\ &= S_1[\psi,\delta\vec{n}]+\int d^dr dz\left[\psi^*\left(-\lambda\vec{h}\cdot\nabla_\perp\right)\psi+\frac{\lambda}{2}\vec{h}\cdot(\psi^*\nabla_\perp\psi-\psi\nabla_\perp\psi^*)\right] \\ &= S_1[\psi,\delta\vec{n}]+\text{(surface terms)}. \end{aligned} \tag{5}$$

Under this symmetry, a term of the form $|\psi|^2\delta\vec{n}^2$ would be forbidden, as a shift in $\delta\vec{n}$ would generate additional terms. However, when the Frank free energy is included in the full action S, it is not invariant under Eq. (4). In particular

$$\begin{aligned} F_n'[\delta\vec{n}'] &= \frac{1}{2}\int d^dr'dz'\left[K_1(\nabla'_\perp\cdot\delta\vec{n}')^2+K_2(\nabla'_\perp\times\delta\vec{n}')^2+K_3(\partial_{z'}\delta\vec{n}')^2\right] \\ &= \frac{1}{2}\int d^dr dz\left[K_1(\nabla_\perp\cdot\delta\vec{n})^2+K_2(\nabla_\perp\times\delta\vec{n})^2+K_3(\partial_z\delta\vec{n}-\lambda\vec{h}\cdot\nabla_\perp\delta\vec{n})^2\right]. \end{aligned} \tag{6}$$

Because the Frank free energy is not invariant, the new relevant operator is generated (see Fig. 1). In the boson analogy, the symmetry above corresponds to the Galilean invariance of the original polymer action (a fully rotationally invariant polymer theory would correspond to a relativistically invariant boson theory). Consider the fully rotationally invariant Frank energy

$$F_n[\mathbf{n}]=\frac{1}{2}\int d^2r\,dz\left[K_1(\nabla\mathbf{n})^2+K_2(\mathbf{n}\cdot(\nabla\mathbf{n}))^2+K_3(\mathbf{n}\times(\nabla\mathbf{n}))^2\right], \tag{7}$$

where boldface represents three-dimensional vectors and $\mathbf{n}$ is a unit vector. We must expand Eq. (7) in powers of $\delta\vec{n}$ so that it too is invariant under Eq. (4). Upon expanding Eq. (7) we arrive at

$$F_n[\delta\vec{n}]\approx\frac{1}{2}\int d^2r\,dz\,\{K_1(\nabla_\perp\cdot\delta\vec{n})^2+K_2(\nabla_\perp\times\delta\vec{n})^2+K_3\left[\partial_z\delta\vec{n}+(\delta\vec{n}\cdot\nabla_\perp)\delta\vec{n}\right]^2\}. \tag{8}$$

The new form for the bend term is not unlike terms required by rotational invariance in smectics [3] and nematic polymers [4]. In fact, in [4], the corrections due to rotational invariance were also irrelevant at one loop and converged in $2+1$ dimensions. We had originally added a factor of λ in the interaction between $\delta\vec{n}$ and ψ to organize perturbation theory. This amounted to rescaling $\delta\vec{n}$ and the Frank constants K_i. Doing this with the new bend term gives

$$F_{\text{bend}}=\frac{1}{2}\int d^dr\,dz\,K_3\left[\partial_z\delta\vec{n}+\lambda(\delta\vec{n}\cdot\nabla_\perp)\delta\vec{n}\right]^2. \tag{9}$$

We have repeated the renormalization-group calculation and find, to one-loop order, that the additional terms in Eq. (9) are irrelevant, and again, the results in [1] are correct.

Though the symmetry prevents $|\psi|^2\delta\vec{n}^2$ from appearing in the action, in perturbation theory this amounts to a delicate cancellation among graphs (see Fig. 2). Checking this explicitly, we find that these graphs do indeed cancel, and our original results are unchanged.

ACKNOWLEDGMENTS

R.D.K. would like to thank M. Goulian for enlightening discussions. R.D.K. was supported in part by the National Science Foundation, through Grant No. PHY92-45317, and through the Ambrose Monell Foundation. P.L.D. was supported by the National Science Foundation, through Grant No. 4-20508. D.R.N. was supported by the National Science Foundation, through Grant No. DMR91-15491, and through the Harvard Materials Research Laboratory.

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